Text Solutions pg. 372—379, #1, 3, 9, 11, 15, 21, 23, 25, 27, 31

1. Expected value.

a) $\mu = E(Y) = 10(0.3) + 20(0.5) + 30(0.2) = 19$

b) $\mu = E(Y) = 2(0.3) + 4(0.4) + 6(0.2) + 8(0.1) = 4.2$

3. Pick a card, any card.

a)

Win	\$0	\$5	\$10	\$30
P(amount won)	$\frac{26}{52}$	$\frac{13}{52}$	$\frac{12}{52}$	$\frac{1}{52}$

b) $\mu = E(\text{amount won}) = \$0\left(\frac{26}{52}\right) + \$5\left(\frac{13}{52}\right) + \$10\left(\frac{12}{52}\right) + \$30\left(\frac{1}{52}\right) \approx \$4.13$

c) Answers may vary. In the long run, the expected payoff of this game is \$4.13 per play. Any amount less than \$4.13 would be a reasonable amount to pay in order to play. Your decision should depend on how long you intend to play. If you are only going to play a few times, you should risk less.

9. Variation 1.

a)

$$\sigma^2 = Var(Y) = (10-19)^2(0.3) + (20-19)^2(0.5) + (30-19)^2(0.2) = 49$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{49} = 7$$

b)

$$\sigma^2 = Var(Y) = (2-4.2)^2(0.3) + (4-4.2)^2(0.4) + (6-4.2)^2(0.2) + (8-4.2)^2(0.1) = 3.56$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{3.56} \approx 1.89$$

Text Solutions pg. 372—379, #1, 3, 9, 11, 15, 21, 23, 25, 27, 31

11. Pick another card.

Answers may vary slightly (due to rounding of the mean)

$$\sigma^2 = Var(\text{Won}) = (0-4.13)^2\left(\frac{26}{52}\right) + (5-4.13)^2\left(\frac{13}{52}\right) + (10-4.13)^2\left(\frac{12}{52}\right) + (30-4.13)^2\left(\frac{1}{52}\right) \approx 29.5396$$

$$\sigma = SD(\text{Won}) = \sqrt{Var(\text{Won})} = \sqrt{29.5396} \approx \$5.44$$

15. Repairs.

a) $\mu = E(\text{Number of Repair Calls}) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 1.7$ calls

b)

$$\sigma^2 = Var(\text{Calls}) = (0-1.7)^2(0.1) + (1-1.7)^2(0.3) + (2-1.7)^2(0.4) + (3-1.7)^2(0.2) = 0.81$$

$$\sigma = SD(\text{Calls}) = \sqrt{Var(\text{Calls})} = \sqrt{0.81} = 0.9 \text{ calls}$$

21. Batteries.

a)

Number good	0	1	2
P(number good)	$\left(\frac{3}{10}\right)\left(\frac{2}{9}\right) = \frac{6}{90}$	$\left(\frac{3}{10}\right)\left(\frac{7}{9}\right) + \left(\frac{7}{10}\right)\left(\frac{3}{9}\right) = \frac{42}{90}$	$\left(\frac{7}{10}\right)\left(\frac{6}{9}\right) = \frac{42}{90}$

b) $\mu = E(\text{number good}) = 0\left(\frac{6}{90}\right) + 1\left(\frac{42}{90}\right) + 2\left(\frac{42}{90}\right) = 1.4$ batteries

c)

$$\sigma^2 = Var(\text{number good}) = (0-1.4)^2\left(\frac{6}{90}\right) + (1-1.4)^2\left(\frac{42}{90}\right) + (2-1.4)^2\left(\frac{42}{90}\right) \approx 0.3733$$

$$\sigma = SD(\text{number good}) = \sqrt{Var(\text{number good})} \approx \sqrt{0.3733} \approx 0.61 \text{ batteries.}$$

Text Solutions pg. 372—379, #1, 3, 9, 11, 15, 21, 23, 25, 27, 31

23. Random variables.

a)
$$\mu = E(3X) = 3(E(X)) = 3(10) = 30$$
$$\sigma = SD(3X) = 3(SD(X)) = 3(2) = 6$$

b)
$$\mu = E(Y+6) = E(Y) + 6 = 20 + 6 = 26$$
$$\sigma = SD(Y+6) = SD(Y) = 5$$

c)
$$\mu = E(X+Y) = E(X) + E(Y) = 10 + 20 = 30$$
$$\sigma = SD(X+Y) = \sqrt{Var(X) + Var(Y)} = \sqrt{2^2 + 5^2} \approx 5.39$$

d)
$$\mu = E(X-Y) = E(X) - E(Y) = 10 - 20 = -10$$
$$\sigma = SD(X-Y) = \sqrt{Var(X) + Var(Y)} = \sqrt{2^2 + 5^2} \approx 5.39$$

e)
$$\mu = E(X_1 + X_2) = E(X) + E(X) = 10 + 10 = 20$$
$$\sigma = SD(X_1 + X_2) = \sqrt{Var(X) + Var(X)} = \sqrt{2^2 + 2^2} \approx 2.83$$

25. Random variables.

a)
$$\mu = E(0.8Y) = 0.8(E(Y)) = 0.8(300) = 240$$
$$\sigma = SD(0.8Y) = 0.8(SD(Y)) = 0.8(16) = 12.8$$

b)
$$\mu = E(2X - 100) = 2(E(X)) - 100 = 140$$
$$\sigma = SD(2X - 100) = 2(SD(X)) = 2(12) = 24$$

c)
$$\mu = E(X + 2Y) = E(X) + 2(E(Y)) = 120 + 2(300) = 720$$
$$\sigma = SD(X + 2Y) = \sqrt{Var(X) + 2^2 Var(Y)} = \sqrt{12^2 + 2^2(16^2)} \approx 34.18$$

d)
$$\mu = E(3X - Y) = 3(E(X)) - E(Y) = 3(120) - 300 = 60$$
$$\sigma = SD(3X - Y) = \sqrt{3^2 Var(X) + Var(Y)} = \sqrt{3^2(12^2) + 16^2} \approx 39.40$$

e)
$$\mu = E(Y_1 + Y_2) = E(Y) + E(Y) = 300 + 300 = 600$$
$$\sigma = SD(Y_1 + Y_2) = \sqrt{Var(Y) + Var(Y)} = \sqrt{16^2 + 16^2} \approx 22.63$$

27. Eggs.

a) $\mu = E(\text{Broken eggs in 3 dozen}) = 3(E(\text{Broken eggs in 1 dozen})) = 3(0.6) = 1.8$ eggs

b) $\sigma = SD(\text{Broken eggs in 3 dozen}) = \sqrt{0.5^2 + 0.5^2 + 0.5^2} \approx 0.87$ eggs

c) The cartons of eggs must be independent of each other.

Text Solutions pg. 372—379, #1, 3, 9, 11, 15, 21, 23, 25, 27, 31

31. Fire!

a) The standard deviation is large because the profits on insurance are highly variable. Although there will be many small gains, there will occasionally be large losses, when the insurance company has to pay a claim.

b)

$$\mu = E(\text{two policies}) = 2(E(\text{one policy})) = 2(150) = \$300$$

$$\sigma = SD(\text{two policies}) = \sqrt{2(Var(\text{one policy}))} = \sqrt{2(6000^2)} \approx \$8,485.28$$

c)

$$\mu = E(10,000 \text{ policies}) = 10,000(E(\text{one policy})) = 10,000(150) = \$1,500,000$$

$$\sigma = SD(10,000 \text{ policies}) = \sqrt{10,000(Var(\text{one policy}))} = \sqrt{10,000(6000^2)} = \$600,000$$

d) If the company sells 10,000 policies, they are likely to be successful. A profit of \$0, is 2.5 standard deviations below the expected profit. This is unlikely to happen. However, if the company sells fewer policies, then the likelihood of turning a profit decreases. In an extreme case, where only two policies are sold, a profit of \$0 is more likely, being only a small fraction of a standard deviation below the mean.

e) This analysis depends on each of the policies being independent from each other. This assumption of independence may be violated if there are many fire insurance claims as a result of a forest fire, or other natural disaster.